form given by (1). The above results are useful for determining the equivalent circuit parameters without the necessity of plotting the curve of ϕ_1 vs ϕ_2 and analyzing this curve by the standard procedures.

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Effect of a Mismatched Ring in a Traveling-Wave Resonant Circuit*

In traveling-wave resonant ring circuits wave amplitude "amplification" has been predicted and shown experimentally.[1-5] In the present note, the input reflection coefficient (input vswr) and wave amplification are considered when the resonant ring circuit contains a mismatch. Qualitatively a small mismatch (low vswr) can produce under resonant conditions a greatly "magnified" input vswr and reduces the maximum attainable amplification in the ring.

In Fig. 1 are shown resonant circuits consisting of ideal lossless directional couplers and a voltage mismatch Γ in each of the lossless waveguide rings. It should be noted that the designation of the output terminals of the directional couplers differ in the two circuits relative to the input terminals. Using scattering matrix notation,[6] the fundamental equations are:

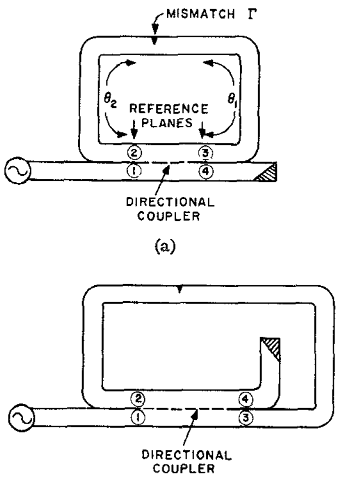

Fig. 1—Resonant ring circuits.

$$\begin{aligned} b_1 &= a_3 S_c \\ b_2 &= a_3 S_T \\ b_3 &= a_1 S_c + a_2 S_T \\ a_2 &= b_3 \tau e^{-j(\theta_1+\theta_2)} + b_2 \Gamma e^{-j2\theta_2} \\ a_3 &= b_3 \Gamma e^{-j2\theta_1} + b_2 \tau e^{-j(\theta_1+\theta_2)} \end{aligned} \tag{1}$$

where

$$S_T \equiv S_{13} = S_{31} = S_{24} = S_{42}$$
$$S_c \equiv S_{14} = S_{41} = S_{23} = S_{32}$$
$$|\tau| = \sqrt{1 - |\Gamma|^2}.$$

The input reflection coefficient, which exhibits a resonance phenomena when ϕ is varied, is given by:

$$\frac{b_1}{a_1} = \frac{S_c^2 \Gamma e^{-j2\theta_1}}{1 + |S_T|^2 e^{-j2\varphi} - 2|\tau S_T| e^{-j\phi}} \tag{2}$$

where

$$\phi \equiv \theta_1 + \theta_2 + \angle S_T + \angle \tau$$
$$\theta_\Gamma - \theta_\tau = \pm \frac{\pi}{2}.$$

If

$$|\Gamma| \le \frac{|S_c|^2}{2 - |S_c|^2},$$

a single-peak resonance occurs when $\phi = 2\pi n$ radians, $n = 1, 2, 3, 4$, etc. and

$$\left|\frac{b_1}{a_1}\right| = \frac{|S_c^2 \Gamma|}{1 + |S_T|^2 - 2|\tau S_T|}. \tag{3}$$

If

$$|\Gamma| \ge \frac{|S_c|^2}{2 - |S_c|^2} \quad \text{then} \quad \left|\frac{b_1}{a_1}\right| = 1$$

when:

$$\phi = \pm \cos^{-1}\left|\frac{\tau(1 + S_T|^2)}{2 S_T}\right|. \tag{4}$$

The two values of ϕ in (4) indicate double-peak resonance behavior.

Eq. (3) is plotted in Fig. 2 and shows that for small values of $|\Gamma|$, there is a reflection coefficient "magnification" of 37.9 and 5.8 for the 10- and 3-db couplers, respectively. For large values of $|\Gamma|$, the input vswr can become infinite at ring resonance, *i.e.*, when (4) is satisfied.

The wave amplification in the ring circuit is given by the following equation and is equivalent to that given by Tischer:[7]

$$\frac{b_3}{a_1} = S_c \frac{1 - |\tau S_T| e^{-j\phi}}{1 + |S_T|^2 e^{-j2\varphi} - 2|\tau S_T| e^{-j\phi}}. \tag{5}$$

Eq. (5) exhibits a single-peak maximum when $\sin\phi = 0$ and a double-peak maxima when

$$\phi = \pm \cos^{-1} \frac{|\tau S_T| + \dfrac{1}{|\tau S_T|} - |\Gamma| \sqrt{\dfrac{1}{|\tau S_T|^2} + |S_T|^2 - 2}}{2}. \tag{6}$$

It should be pointed out that for very small values of $|\Gamma|$, only a single-peak resonance exists since $\cos\phi$ of (6) exceeds unity. The maximum values of $|\Gamma|$ which will yield single-peak resonance for 10-db and 3-db

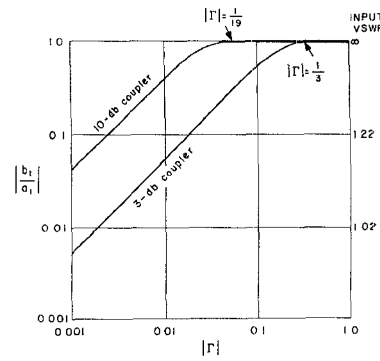

Fig. 2—Input reflection coefficient.

couplers are 0.026 and 0.177 respectively. To a very rough approximation these values of $|\Gamma|$ are about one-half of $|S_c|^2/(2-|S_c^2|)$. Operating under single-peak resonance conditions, the wave amplification $|b_3/a_1|$ as a function of $|\Gamma|$ is plotted in Fig. 3.

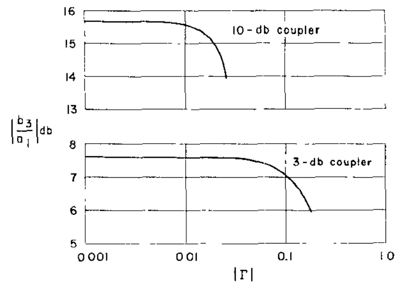

Fig. 3—Wave "amplification."

Another effect within the resonant ring due to the mismatch is on the ratio of incoming to outgoing waves at terminal 3 in Fig. 1. The analysis shows that with a 3-db coupler and when $\sin\phi = 0$,

$$\left|\frac{a_3}{b_3}\right| = \frac{\sqrt{2}|\Gamma|}{\sqrt{2} - |\tau|} \tag{7}$$
$$\doteq 3.42\,|\Gamma| \text{ for small values of } |\Gamma|.$$

Thus the vswr within the ring circuit is also "magnified." For a 10-db coupler the corresponding quantity is $19.5|\Gamma|$.

Referring to the annular-waveguide rotary joint,[2] the input vswr had a maximum value of about 3 (diagonal-arm condition) when the coupler "*A*" was about half open or approximately at 3-db coupling. These data would imply, referring to Fig. 2 above, that the mismatch vswr in the ring would be about 1.2 and this value appears entirely plausible.

In summary, a traveling-wave resonator requires very careful matching in the ring circuit if the input vswr is to be low and if maximum wave amplification is desired.

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